Background knowledge

Contents:

This section contains material that is normally covered prior to this course. It is assumed, background knowledge. Not all preliminaries are covered within it. However, other necessary work is revised within the chapters which follow this one.

A OPERATIONS WITH SURDS (RADICALS)

Real numbers like $\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$, $\sqrt{6}$, etc., are called **surds** or **radicals**. Surds are present in solutions to some quadratic equations. $\sqrt{4}$ is not a surd as it simplifies to 2.

Definition: $\sqrt{a}$ is the non-negative number such that $\sqrt{a} \times \sqrt{a} = a$.

Properties:

- $\sqrt{a}$ is never negative, that is, $\sqrt{a} \geqslant 0$.
- $\sqrt{a}$ is meaningful only for $a \geqslant 0$.
- $\sqrt{ab} = \sqrt{a} \times \sqrt{b}$ for $a \geqslant 0$ and $b \geqslant 0$.
- $\sqrt{\dfrac{a}{b}} = \dfrac{\sqrt{a}}{\sqrt{b}}$ for $a \geqslant 0$ and $b > 0$.

SURDIC OPERATIONS

Example 1

Write as a single surd: **a** $\sqrt{2} \times \sqrt{3}$ **b** $\dfrac{\sqrt{18}}{\sqrt{6}}$

a $\sqrt{2} \times \sqrt{3}$
$= \sqrt{2 \times 3}$
$= \sqrt{6}$

b $\dfrac{\sqrt{18}}{\sqrt{6}}$
$= \sqrt{\dfrac{18}{6}}$
$= \sqrt{3}$

or $\dfrac{\sqrt{18}}{\sqrt{6}}$
$= \dfrac{\sqrt{6} \times \sqrt{3}}{\sqrt{6}}$
$= \sqrt{3}$

EXERCISE A

1 Write as a single surd or rational number:

a $\sqrt{3} \times \sqrt{5}$ **b** $(\sqrt{3})^2$ **c** $2\sqrt{2} \times \sqrt{2}$ **d** $3\sqrt{2} \times 2\sqrt{2}$

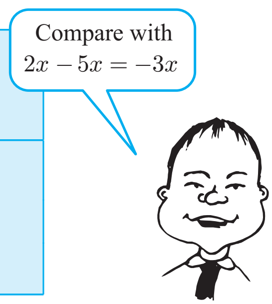
e $3\sqrt{7} \times 2\sqrt{7}$ **f** $\dfrac{\sqrt{12}}{\sqrt{2}}$ **g** $\dfrac{\sqrt{12}}{\sqrt{6}}$ **h** $\dfrac{\sqrt{18}}{\sqrt{3}}$

Example 2

Simplify: **a** $3\sqrt{3} + 5\sqrt{3}$ **b** $2\sqrt{2} - 5\sqrt{2}$

a $3\sqrt{3} + 5\sqrt{3}$
$= (3 + 5)\sqrt{3}$
$= 8\sqrt{3}$

b $2\sqrt{2} - 5\sqrt{2}$
$= (2 - 5)\sqrt{2}$
$= -3\sqrt{2}$



2 Simplify the following mentally:

a $2\sqrt{2}+3\sqrt{2}$	**b** $2\sqrt{2}-3\sqrt{2}$	**c** $5\sqrt{5}-3\sqrt{5}$	**d** $5\sqrt{5}+3\sqrt{5}$
e $3\sqrt{5}-5\sqrt{5}$	**f** $7\sqrt{3}+2\sqrt{3}$	**g** $9\sqrt{6}-12\sqrt{6}$	**h** $\sqrt{2}+\sqrt{2}+\sqrt{2}$

Example 3

Write $\sqrt{18}$ in the form $a\sqrt{b}$ where a and b are integers, a is as large as possible.

$\sqrt{18}$

$= \sqrt{9\times 2}$ {9 is the largest perfect square factor of 18}

$= \sqrt{9}\times\sqrt{2}$

$= 3\sqrt{2}$

3 Write the following in the form $a\sqrt{b}$ where a and b are integers and a is as large as possible:

a $\sqrt{8}$	**b** $\sqrt{12}$	**c** $\sqrt{20}$	**d** $\sqrt{32}$
e $\sqrt{27}$	**f** $\sqrt{45}$	**g** $\sqrt{48}$	**h** $\sqrt{54}$
i $\sqrt{50}$	**j** $\sqrt{80}$	**k** $\sqrt{96}$	**l** $\sqrt{108}$

Example 4

Simplify: $2\sqrt{75}-5\sqrt{27}$

$2\sqrt{75}-5\sqrt{27}$

$= 2\sqrt{25\times 3} - 5\sqrt{9\times 3}$

$= 2\times 5\times\sqrt{3} - 5\times 3\times\sqrt{3}$

$= 10\sqrt{3}-15\sqrt{3}$

$= -5\sqrt{3}$

4 Simplify:

a $4\sqrt{3}-\sqrt{12}$	**b** $3\sqrt{2}+\sqrt{50}$	**c** $3\sqrt{6}+\sqrt{24}$
d $2\sqrt{27}+2\sqrt{12}$	**e** $\sqrt{75}-\sqrt{12}$	**f** $\sqrt{2}+\sqrt{8}-\sqrt{32}$

Example 5

Write $\frac{9}{\sqrt{3}}$ without a radical in the denominator.

$\frac{9}{\sqrt{3}}$

$= \frac{9}{\sqrt{3}}\times\frac{\sqrt{3}}{\sqrt{3}}$

$= \frac{9\sqrt{3}}{3}$

$= 3\sqrt{3}$

5 Write without a radical in the denominator:

a $\frac{1}{\sqrt{2}}$ **b** $\frac{6}{\sqrt{3}}$ **c** $\frac{7}{\sqrt{2}}$ **d** $\frac{10}{\sqrt{5}}$ **e** $\frac{10}{\sqrt{2}}$

f $\frac{18}{\sqrt{6}}$ **g** $\frac{12}{\sqrt{3}}$ **h** $\frac{5}{\sqrt{7}}$ **i** $\frac{14}{\sqrt{7}}$ **j** $\frac{2\sqrt{3}}{\sqrt{2}}$

B STANDARD FORM (SCIENTIFIC NOTATION)

Standard form (or **scientific notation**) involves writing any given number as *a number between* 1 *and* 10, multiplied by a *power of* 10,

i.e., $a \times 10^n$ where a lies between 1 and 10.

Example 6

Write in standard form: **a** 37 600 **b** 0.000 86

a $37\,600 = 3.76 \times 10\,000$ {shift decimal point 4 places to the left and $\times$ 10 000}

$= 3.76 \times 10^4$

b $0.000\,86 = 8.6 \div 10^4$ {shift decimal point 4 places to the right and $\div$ 10 000}

$= 8.6 \times 10^{-4}$

EXERCISE B

1 Express the following in standard form (scientific notation):

a 259 **b** 259 000 **c** 2.59 **d** 0.259

e 0.000 259 **f** 40.7 **g** 4070 **h** 0.0407

i 407 000 **j** 407 000 000 **k** 0.000 040 7

2 Express the following in standard form (scientific notation):

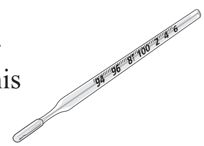
a The distance from the Earth to the Sun is 149 500 000 000 m.

b Bacteria are single cell organisms, some of which have a diameter of 0.000 3 mm.

c A speck of dust is smaller than 0.001 mm.

d The central temperature of the Sun is 15 million degrees Celsius.

e A single red blood cell lives for about four months and during this time it will circulate around the body 300 000 times.



Example 7

Write as an ordinary number:

a 3.2×10^2 **b** 5.76×10^{-5}

a 3.2×10^2

$= 3.20 \times 100$

$= 320$

b 5.76×10^{-5}

$= 000005.76 \div 10^5$

$= 0.000\,057\,6$

3 Write as an ordinary decimal number:

a 4×10^3	**b** 5×10^2	**c** 2.1×10^3	**d** 7.8×10^4
e 3.8×10^5	**f** 8.6×10^1	**g** 4.33×10^7	**h** 6×10^7

4 Write as an ordinary decimal number:

a 4×10^{-3}	**b** 5×10^{-2}	**c** 2.1×10^{-3}	**d** 7.8×10^{-4}
e 3.8×10^{-5}	**f** 8.6×10^{-1}	**g** 4.33×10^{-7}	**h** 6×10^{-7}

5 Write as an ordinary decimal number:

a The wave length of light is 9×10^{-7} m.

b The estimated world population for the year 2000 is 6.130×10^9.

c The diameter of our galaxy, the Milky Way, is 1×10^5 light years.

d The smallest viruses are 1×10^{-5} mm in size.

6 Find, with decimal part correct to 2 places:

a $(3.42\times10^5)\times(4.8\times10^4)$	**b** $(6.42 \times 10^{-2})^2$	**c** $\dfrac{3.16 \times 10^{-10}}{6 \times 10^7}$
d $(9.8 \times 10^{-4}) \div (7.2 \times 10^{-6})$	**e** $\dfrac{1}{3.8 \times 10^5}$	**f** $(1.2 \times 10^3)^3$

7 If a missile travels at 5400 km/h how far will it travel in:

a 1 day **b** 1 week **c** 2 years?

[Give your answers in standard form with decimal part correct to 2 places and assume that 1 year $\doteqdot$ 365.25 days.]

8 Light travels at a speed of 3×10^8 metres per second. How far will light travel in:

a 1 minute **b** 1 day **c** 1 year?

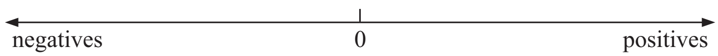
[Give your answers with decimal part correct to 2 decimal places and assume that 1 year $\doteqdot$ 365.25 days.]

C NUMBER SYSTEMS AND SET NOTATION

NUMBER SYSTEMS

We will use

- $\mathcal{R}$ to represent the set of all **real numbers**. These are all the numbers on the number line.



- **N** to represent the set of all **natural numbers**. $\mathbf{N} = \{0, 1, 2, 3, 4, 5, ...\}$
- **Z** to represent the set of all **integers**. $\mathbf{Z} = \{0, \pm1, \pm2, \pm3, \pm4, ...\}$
  Note: $\mathbf{Z}^+$ is the set of all positive integers. $\mathbf{Z}^+ = \{1, 2, 3, 4, ...\}$
- **Q** to represent the set of all **rational numbers** which are any numbers of the form $\dfrac{p}{q}$ where p and q are integers, $q \neq 0$.

SET NOTATION

$\{x: \ -3 < x < 2\}$ reads "the set of all values that x can be such that x lies between -3 and 2".

the set of all (points to $\{$) *such that* (points to $:$)

EXERCISE C

1 Write verbal statements for the meaning of:

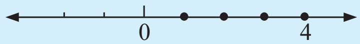
a $\{x: \ x > 5\}$ **b** $\{x: \ x \leqslant 3\}$ **c** $\{y: \ 0 < y < 6\}$

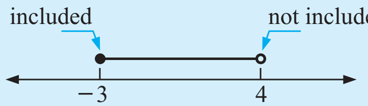
d $\{x: \ 2 \leqslant x \leqslant 4\}$ **e** $\{t: \ 1 < t < 5\}$ **f** $\{n: \ n < 2 \ \text{ or } \ n \geqslant 6\}$

Note: If a number set like **N**, **Z** or **Q** is not given we assume we are referring to real numbers (i.e., in $\mathcal{R}$).

Example 8

Write in set notation:

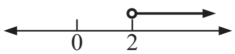
a 

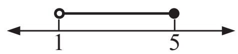
b 

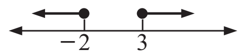
a $\{x: \ x \in \mathbf{N}, \ \ 1 \leqslant x \leqslant 4\}$
or $\{x: \ x \in \mathbf{Z}, \ 1 \leqslant x \leqslant 4\}$

b $\{x: \ -3 \leqslant x < 4\}$
Note: $\in$ is used to mean "is in"

2 Write in set notation:

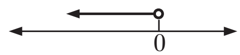
a 

b 

c 

d 

e 

f 

3 Sketch the following number sets:

a $\{x: \ x \in \mathbf{N}, \ \ 4 \leqslant x < 10\}$ **b** $\{x: \ x \in \mathbf{Z}, \ \ -4 < x \leqslant 5\}$

c $\{x: \ x \in \mathcal{R}, \ \ -5 < x \leqslant 4\}$ **d** $\{x: \ x \in \mathbf{Z}, \ \ x > -4\}$

e $\{x: \ x \in \mathcal{R}, \ \ x \leqslant 8\}$

D ALGEBRAIC SIMPLIFICATION

Recall that $\boldsymbol{a(b+c) = ab + ac}$ and $\boldsymbol{a(b-c) = ab - ac}$

EXERCISE D

1 Simplify if possible:

a $3x + 7x - 10$ **b** $3x + 7x - x$ **c** $2x + 3x + 5y$

d $8 - 6x - 2x$ **e** $7ab + 5ba$ **f** $3x^2 + 7x^3$

2 Remove brackets and then simplify:

a $3(2x+5) + 4(5+4x)$ **b** $6 - 2(3x-5)$

c $5(2a-3b) - 6(a-2b)$ **d** $3x(x^2 - 7x + 3) - (1 - 2x - 5x^2)$

3 Simplify:

a $2x(3x)^2$ **b** $\frac{3a^2b^3}{9ab^4}$ **c** $\sqrt{16x^4}$ **d** $(2a^2)^3 \times 3a^4$

E LINEAR EQUATIONS AND INEQUALITIES

EXERCISE E

Reminder: Multiplying or dividing both sides by a negative reverses the inequality sign.

1 Solve for x:

a $2x + 5 = 25$ **b** $3x - 7 > 11$ **c** $5x + 16 = 20$

d $\frac{x}{3} - 7 = 10$ **e** $6x + 11 < 4x - 9$ **f** $\frac{3x - 2}{5} = 8$

g $1 - 2x \geqslant 19$ **h** $\frac{1}{2}x + 1 = \frac{2}{3}x - 2$ **i** $\frac{2}{3} - \frac{3x}{4} = \frac{1}{2}(2x - 1)$

2 Solve simultaneously for x and y:

a $x + 2y = 9$, $x - y = 3$ **b** $2x + 5y = 28$, $x - 2y = 2$ **c** $7x + 2y = -4$, $3x + 4y = 14$

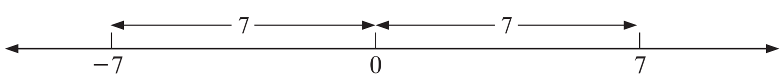
d $5x - 4y = 27$, $3x + 2y = 9$ **e** $x + 2y = 5$, $2x + 4y = 1$ **f** $\frac{x}{2} + \frac{y}{3} = 5$, $\frac{x}{3} + \frac{y}{4} = 1$

F ABSOLUTE VALUE (MODULUS)

The **modulus** (**absolute value**) of a real number is its size, ignoring its sign.

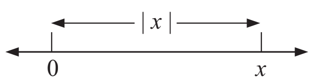
For example: the modulus (or absolute value) of 7 is 7, and
the modulus (or absolute value) of −7 is also 7.

Geometrically, the modulus of a real number can be interpreted as its *distance* from zero (0) on the number line. Because the modulus is distance, it cannot be negative.

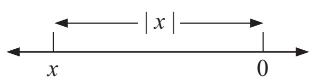


Thus, $|x|$ is the distance of x from 0 on the number line.

If $x > 0$



If $x < 0$



ALGEBRAIC DEFINITION

$$|x| = \begin{cases} x & \text{if } x \geqslant 0 \\ -x & \text{if } x < 0 \end{cases} \quad \text{or} \quad |x| = \sqrt{x^2}$$

EXERCISE F

1 Find the value of:

- **a** $5-(-11)$
- **b** $|5|-|-11|$
- **c** $|5-(-11)|$
- **d** $\left|(-2)^2+11(-2)\right|$
- **e** $|-6|-|-8|$
- **f** $|-6-(-8)|$

2 If $a=-2,\ b=3,\ c=-4$ find the value of:

- **a** $|a|$
- **b** $|b|$
- **c** $|a|\,|b|$
- **d** $|ab|$
- **e** $|a-b|$
- **f** $|a|-|b|$
- **g** $|a+b|$
- **h** $|a|+|b|$
- **i** $|a|^2$
- **j** a^2
- **k** $\left|\dfrac{c}{a}\right|$
- **l** $\dfrac{|c|}{|a|}$

MODULUS EQUATIONS

It is clear that $|x|=2$ has two solutions, $x=2$ and $x=-2$.

In general, **if $|x|=a$ where $a>0$, then $x=\pm a$.**

3 Solve for x:

- **a** $|x|=3$
- **b** $|x|=-5$
- **c** $|x|=0$
- **d** $|x-1|=3$
- **e** $|3-x|=4$
- **f** $|x+5|=-1$
- **g** $|3x-2|=1$
- **h** $|3-2x|=3$
- **i** $|2-5x|=12$

G PRODUCT EXPANSION

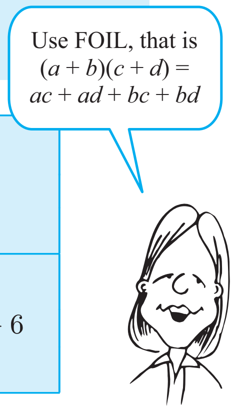
$y=2(x-1)(x+3)$ can be expanded into the general form $y=ax^2+bx+c$.

Likewise, $y=2(x-3)^2+7$ can be expanded into this from.

We will review expansion techniques.

Following is a **list of expansion rules** you should use:

- $(a+b)(c+d)=ac+ad+bc+bd$ {sometimes called the **FOIL** rule}
- $(a+b)(a-b)=a^2-b^2$ {**difference of two squares**}
- $(a+b)^2=a^2+2ab+b^2$, $(a-b)^2=a^2-2ab+b^2$ {**perfect squares**}



Example 9

Expand and simplify:

a $(2x+1)(x+3)$ **b** $(3x-2)(x+3)$

a
$$\begin{aligned}&(2x+1)(x+3)\\&=2x^2+6x+x+3\\&=2x^2+7x+3\end{aligned}$$

b
$$\begin{aligned}&(3x-2)(x+3)\\&=3x^2+9x-2x-6\\&=3x^2+7x-6\end{aligned}$$

EXERCISE G

1 Expand and simplify using $(a+b)(c+d) = ac + ad + bc + bd$:

a $(2x+3)(x+1)$	**b** $(3x+4)(x+2)$	**c** $(5x-2)(2x+1)$
d $(x+2)(3x-5)$	**e** $(7-2x)(2+3x)$	**f** $(1-3x)(5+2x)$
g $(3x+4)(5x-3)$	**h** $(1-3x)(2-5x)$	**i** $(7-x)(3-2x)$
j $(5-2x)(3-2x)$	**k** $-(x+1)(x+2)$	**l** $-2(x-1)(2x+3)$

Example 10

Expand using the rule $(a+b)(a-b) = a^2 - b^2$:

a $(5x-2)(5x+2)$ **b** $(7+2x)(7-2x)$

Remember that $(a+b)(a-b) = a^2 - b^2$

a $(5x-2)(5x+2)$
$= (5x)^2 - 2^2$
$= 25x^2 - 4$

b $(7+2x)(7-2x)$
$= 7^2 - (2x)^2$
$= 49 - 4x^2$

2 Expand using the rule $(a+b)(a-b) = a^2 - b^2$:

a $(x+6)(x-6)$	**b** $(x+8)(x-8)$	**c** $(2x-1)(2x+1)$
d $(3x-2)(3x+2)$	**e** $(4x+5)(4x-5)$	**f** $(5x-3)(5x+3)$
g $(3-x)(3+x)$	**h** $(7-x)(7+x)$	**i** $(7+2x)(7-2x)$
j $(x+\sqrt{2})(x-\sqrt{2})$	**k** $(x+\sqrt{5})(x-\sqrt{5})$	**l** $(2x-\sqrt{3})(2x+\sqrt{3})$

Example 11

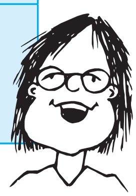
Use $(a+b)^2 = a^2 + 2ab + b^2$ or $(a-b)^2 = a^2 - 2ab + b^2$

Expand using perfect square expansion rules:

a $(x+2)^2$ **b** $(3x-1)^2$

a $(x+2)^2$
$= x^2 + 2(x)(2) + 2^2$
$= x^2 + 4x + 4$

b $(3x-1)^2$
$= (3x)^2 - 2(3x)(1) + 1^2$
$= 9x^2 - 6x + 1$



3 Expand and simplify using the perfect square expansion rules:

a $(x+5)^2$	**b** $(x+7)^2$	**c** $(x-2)^2$	**d** $(x-6)^2$
e $(3+x)^2$	**f** $(5+x)^2$	**g** $(11-x)^2$	**h** $(10-x)^2$
i $(2x+7)^2$	**j** $(3x+2)^2$	**k** $(5-2x)^2$	**l** $(7-3x)^2$

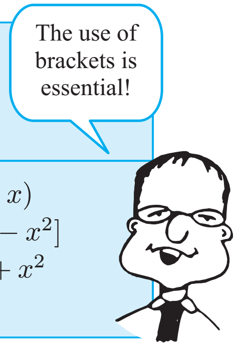
4 Expand the following into the general form $y = ax^2 + bx + c$:

a $y = 2(x+2)(x+3)$	**b** $y = 3(x-1)^2 + 4$	**c** $y = -(x+1)(x-7)$
d $y = -(x+2)^2 - 11$	**e** $y = 4(x-1)(x-5)$	**f** $y = -\frac{1}{2}(x+4)^2 - 6$
g $y = -5(x-1)(x-6)$	**h** $y = \frac{1}{2}(x+2)^2 - 6$	**i** $y = -\frac{5}{2}(x-4)^2$

Example 12

Expand and simplify:

a $1 - 2(x+3)^2$ **b** $2(3+x) - (2+x)(3-x)$



a
$$\begin{aligned} &1 - 2(x+3)^2 \\ &= 1 - 2[x^2 + 6x + 9] \\ &= 1 - 2x^2 - 12x - 18 \\ &= -2x^2 - 12x - 17 \end{aligned}$$

b
$$\begin{aligned} &2(3+x) - (2+x)(3-x) \\ &= 6 + 2x - [6 - 2x + 3x - x^2] \\ &= 6 + 2x - 6 + 2x - 3x + x^2 \\ &= x^2 + x \end{aligned}$$

5 Expand and simplify:

a $1 + 2(x+3)^2$
b $2 + 3(x-2)(x+3)$
c $3 - (3-x)^2$
d $5 - (x+5)(x-4)$
e $1 + 2(4-x)^2$
f $x^2 - 3x - (x+2)(x-2)$
g $(x+2)^2 - (x+1)(x-4)$
h $(2x+3)^2 + 3(x+1)^2$
i $x^2 + 3x - 2(x-4)^2$
j $(3x-2)^2 - 2(x+1)^2$

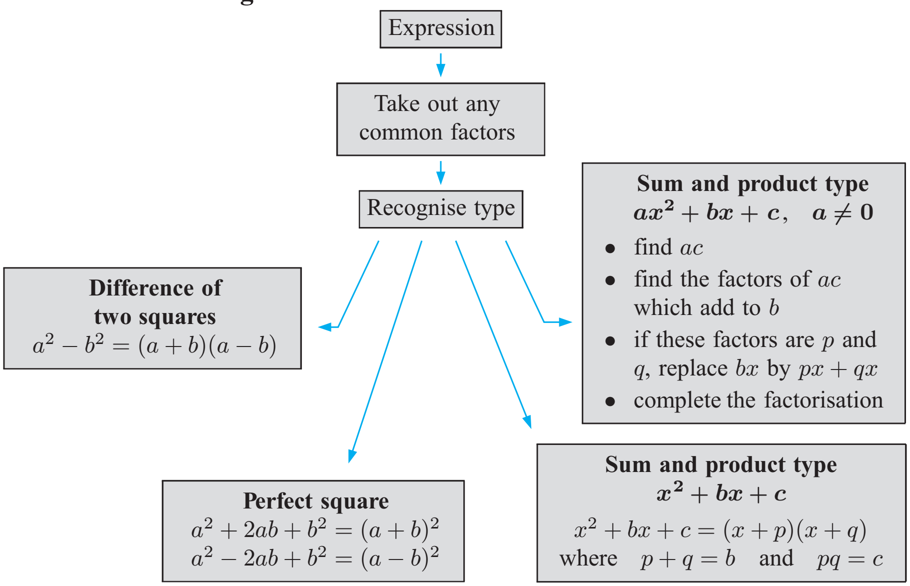
H FACTORISATION

Algebraic **factorisation** is the reverse process of expansion.

For example, $(2x+1)(x-3)$ is **expanded** to $2x^2 - 5x - 3$, whereas
$2x^2 - 5x - 3$ is **factorised** to $(2x+1)(x-3)$.

Notice that $2x^2 - 5x - 3 = (2x+1)(x-3)$ has been factorised into two **linear factors**.

Flow chart for factorising:



Example 13

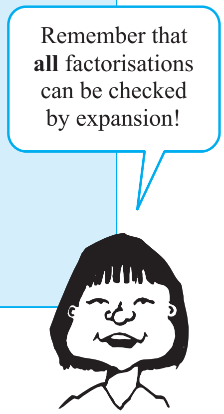
Fully factorise:

a $3x^2 - 12x$ **b** $4x^2 - 1$ **c** $x^2 - 12x + 36$

a $3x^2 - 12x$ {has $3x$ as common factor}
$= 3x(x - 4)$

b $4x^2 - 1$
$= (2x)^2 - 1^2$ {difference of two squares}
$= (2x + 1)(2x - 1)$

c $x^2 - 12x + 36$
$= x^2 - 2(x)(6) + 6^2$ {perfect square form}
$= (x - 6)^2$



EXERCISE H

1 Fully factorise:

a $3x^2 + 9x$	**b** $2x^2 + 7x$	**c** $4x^2 - 10x$
d $6x^2 - 15x$	**e** $9x^2 - 25$	**f** $16x^2 - 1$
g $2x^2 - 8$	**h** $3x^2 - 9$	**i** $4x^2 - 20$
j $x^2 - 8x + 16$	**k** $x^2 - 10x + 25$	**l** $2x^2 - 8x + 8$
m $16x^2 + 40x + 25$	**n** $9x^2 + 12x + 4$	**o** $x^2 - 22x + 121$

Example 14

Fully factorise:

a $3x^2 + 12x + 9$ **b** $-x^2 + 3x + 10$

a $3x^2 + 12x + 9$ {has 3 as a common factor}
$= 3(x^2 + 4x + 3)$ {so, sum $= 4$, product $= 3$}
$= 3(x + 1)(x + 3)$

b $-x^2 + 3x + 10$
$= -[x^2 - 3x - 10]$ {removing -1 as common factor to make coefficient of x^2 be 1}
$= -(x - 5)(x + 2)$ {as sum $= -3$, product $= -10$}

2 Fully factorise:

a $x^2 + 9x + 8$	**b** $x^2 + 7x + 12$	**c** $x^2 - 7x - 18$
d $x^2 + 4x - 21$	**e** $x^2 - 9x + 18$	**f** $x^2 + x - 6$
g $-x^2 + x + 2$	**h** $3x^2 - 42x + 99$	**i** $-2x^2 - 4x - 2$
j $2x^2 + 6x - 20$	**k** $2x^2 - 10x - 48$	**l** $-2x^2 + 14x - 12$
m $-3x^2 + 6x - 3$	**n** $-x^2 - 2x - 1$	**o** $-5x^2 + 10x + 40$

FACTORISATION BY 'SPLITTING' THE x-TERM

Using the distributive law to expand we see that:

$$\begin{aligned}&(2x+3)(4x+5)\\ &= 8x^2+10x+12x+15\\ &= 8x^2+22x+15\end{aligned}$$

We will now **reverse** the process to **factorise** the quadratic expression $8x^2+22x+15$.

Notice that:		$8x^2+22x+15$
Step 1:	Split the middle term	$= 8x^2+10x+12x+15$
Step 2:	Group in pairs	$= (8x^2+10x)+(12x+15)$
Step 3:	Factorise each pair separately	$= 2x(4x+5)+3(4x+5)$
Step 4:	Factorise fully	$= (4x+5)(2x+3)$

The "trick" in factorising these types of quadratic expressions is in *Step 1* where the middle term needs to be split into two so that the rest of the factorisation proceeds smoothly.

Rules for splitting the x-term:

The following procedure is recommended for factorising ax^2+bx+c:

- find ac
- find the factors of ac which add to b
- if these factors are p and q replace bx by $px+qx$
- complete the factorisation.

Example 15

Fully factorise:

a $2x^2-x-10$ **b** $6x^2-25x+14$

a $2x^2-x-10$ has $ac = 2\times -10 = -20$.
The factors of -20 which add to -1 are -5 and $+4$.

$$\begin{aligned}&= 2x^2-5x+4x-10\\ &= x(2x-5)+2(2x-5)\\ &= (2x-5)(x+2)\end{aligned}$$

b $6x^2-25x+14$ has $ac = 6\times 14 = 84$.
The factors of 84 which add to -25 are -21 and -4.

$$\begin{aligned}&= 6x^2-21x-4x+14\\ &= 3x(2x-7)-2(2x-7)\\ &= (2x-7)(3x-2)\end{aligned}$$

3 Fully factorise:

a $2x^2+5x-12$
b $3x^2-5x-2$
c $7x^2-9x+2$
d $6x^2-x-2$
e $4x^2-4x-3$
f $10x^2-x-3$
g $2x^2-11x-6$
h $3x^2-5x-28$
i $8x^2+2x-3$
j $10x^2-9x-9$
k $3x^2+23x-8$
l $6x^2+7x+2$
m $-4x^2-2x+6$
n $12x^2-16x-3$
o $-6x^2-9x+42$
p $21x-10-9x^2$
q $8x^2-6x-27$
r $12x^2+13x+3$
s $12x^2+20x+3$
t $15x^2-22x+8$
u $14x^2-11x-15$

Example 16

Fully factorise: $3(x+2)+2(x-1)(x+2)-(x+2)^2$

$$\begin{aligned} &3(x+2)+2(x-1)(x+2)-(x+2)^2 \\ &=(x+2)[3+2(x-1)-(x+2)] \quad \{\text{as } (x+2) \text{ is the common factor}\} \\ &=(x+2)[3+2x-2-x-2] \\ &=(x+2)(x-1) \end{aligned}$$

4 Fully factorise:

a $3(x+4)+2(x+4)(x-1)$
b $8(2-x)-3(x+1)(2-x)$
c $6(x+2)^2+9(x+2)$
d $4(x+5)+8(x+5)^2$
e $(x+2)(x+3)-(x+3)(2-x)$
f $(x+3)^2+2(x+3)-x(x+3)$
g $5(x-2)-3(2-x)(x+7)$
h $3(1-x)+2(x+1)(x-1)$

INVESTIGATION ANOTHER FACTORISATION TECHNIQUE

What to do:

1 By expanding, show that $\dfrac{(ax+p)(ax+q)}{a}=ax^2+[p+q]x+\left[\dfrac{pq}{a}\right]$.

2 If $ax^2+bx+c=\dfrac{(ax+p)(ax+q)}{a}$, show that $p+q=b$ and $pq=ac$.

3 Using **2** on $8x^2+22x+15$, we have

$$8x^2+22x+15=\frac{(8x+p)(8x+q)}{8} \quad \text{where} \quad \begin{cases} p+q=22 \\ pq=8\times 15=120 \end{cases}$$

So, $p=12$, $q=10$ (or vice versa)

$$\begin{aligned} \therefore \quad 8x^2+22x+15 &= \frac{(8x+12)(8x+10)}{8} \\ &= \frac{\cancel{4}(2x+3)\cancel{2}(4x+5)}{\cancel{8}} \\ &= (2x+3)(4x+5) \end{aligned}$$

a Use the method shown to factorise:

i $3x^2 + 14x + 8$ **ii** $12x^2 + 17x + 6$ **iii** $15x^2 + 14x - 8$

b Check your answers to **a** using expansion.

Example 17

Fully factorise using the 'difference of two squares':

a $(x+2)^2 - 9$ **b** $(1-x)^2 - (2x+1)^2$

a
$$\begin{aligned} &(x+2)^2 - 9 \\ &= (x+2)^2 - 3^2 \\ &= [(x+2)+3][(x+2)-3] \\ &= (x+5)(x-1) \end{aligned}$$

b
$$\begin{aligned} &(1-x)^2 - (2x+1)^2 \\ &= [(1-x)-(2x+1)][(1-x)+(2x+1)] \\ &= [1-x-2x-1][1-x+2x+1] \\ &= -3x(x+2) \end{aligned}$$

5 Fully factorise:

a $(x+3)^2 - 16$ **b** $4 - (1-x)^2$ **c** $(x+4)^2 - (x-2)^2$

d $16 - 4(x+2)^2$ **e** $(2x+3)^2 - (x-1)^2$ **f** $(x+h)^2 - x^2$

g $3x^2 - 3(x+2)^2$ **h** $5x^2 - 20(2-x)^2$ **i** $12x^2 - 27(3+x)^2$

I FORMULA REARRANGEMENT

For the formula $D = xt + p$ we say that D is the **subject**. This is because D is expressed in terms of the other variables, x, t and p.

In formula rearrangement we require one of the other variables to be the subject.

To **rearrange** a formula we use the same processes as used for solving an equation for the variable we wish to be the subject.

Example 18

Make x the subject of $D = xt + p$.

$$\begin{aligned} \text{If} \quad D &= xt + p \\ \text{then} \quad xt + p &= D \\ \therefore \quad xt + p - p &= D - p && \{\text{subtract } p \text{ from both sides}\} \\ \therefore \quad xt &= D - p \\ \therefore \quad \frac{xt}{t} &= \frac{D-p}{t} && \{\text{divide both sides by } t\} \\ \therefore \quad x &= \frac{D-p}{t} \end{aligned}$$

EXERCISE I

1 Make x the subject of:

a $a+x=b$ **b** $ax=b$ **c** $2x+a=d$

d $c+x=t$ **e** $5x+2y=20$ **f** $2x+3y=12$

g $7x+3y=d$ **h** $ax+by=c$ **i** $y=mx+c$

Example 19

Make z the subject of $c=\dfrac{m}{z}$.

$$c=\frac{m}{z}$$

$$c\times z=\frac{m}{z}\times z \qquad \{\text{multiply both sides by } z\}$$

$$\therefore\ cz=m$$

$$\therefore\ \frac{cz}{c}=\frac{m}{c} \qquad \{\text{divide both sides by } c\}$$

$$\therefore\ z=\frac{m}{c}$$

2 Make z the subject of:

a $az=\dfrac{b}{c}$ **b** $\dfrac{a}{z}=d$ **c** $\dfrac{3}{d}=\dfrac{2}{z}$

3 Make:

a a the subject of $F=ma$ **b** r the subject of $C=2\pi r$

c d the subject of $V=ldh$ **d** K the subject of $A=\dfrac{b}{K}$

Example 20

Make t the subject of $s=\frac{1}{2}gt^2$ where $t>0$.

$$\tfrac{1}{2}gt^2=s \qquad \{\text{rewrite with } t^2 \text{ on LHS}\}$$

$$\therefore\ 2\times\tfrac{1}{2}gt^2=2\times s \qquad \{\text{multiply both sides by } 2\}$$

$$\therefore\ gt^2=2s$$

$$\therefore\ \frac{gt^2}{g}=\frac{2s}{g} \qquad \{\text{divide both sides by } g\}$$

$$\therefore\ t^2=\frac{2s}{g}$$

$$\therefore\ t=\sqrt{\frac{2s}{g}} \qquad \{\text{as } t>0\}$$

4 Make:

a r the subject of $A = \pi r^2$, $(r > 0)$

b x the subject of $N = \dfrac{x^5}{a}$

c r the subject of $V = \frac{4}{3}\pi r^3$

d x the subject of $D = \dfrac{n}{x^3}$

5 Make:

a a the subject of $d = \dfrac{\sqrt{a}}{n}$

b l the subject of $T = \frac{1}{5}\sqrt{l}$

c a the subject of $c = \sqrt{a^2 - b^2}$

d l the subject of $T = 2\pi\sqrt{\dfrac{l}{g}}$

e a the subject of $P = 2(a + b)$

f h the subject of $A = \pi r^2 + 2\pi rh$

g r the subject of $I = \dfrac{E}{R + r}$

h q the subject of $A = \dfrac{B}{p - q}$

6 **a** Given the formula $k = \dfrac{d^2}{2ab}$, make a the subject of the formula.

b Find the value for a when $k = 112$, $d = 24$, $b = 2$.

7 The formula for determining the volume of a sphere is $V = \frac{4}{3}\pi r^3$ where r is the radius.

a Make r the subject of the formula.

b Find the radius of a sphere having a volume of 40 cm^3.

8 The distance (S cm) travelled by an object accelerating from a stationary position is given by the formula $S = \frac{1}{2}at^2$ where a is the acceleration (cm/sec^2) and t is the time (seconds).

a Make t the subject of the formula. (Consider only $t > 0$.)

b Find the time taken for an object accelerating at 8 cm/sec^2 to travel 10 m.

9 The relationship between object and image distances (in cm) for a concave mirror can be written as $\dfrac{1}{f} = \dfrac{1}{u} + \dfrac{1}{v}$ where f is the focal length, u is the object distance and v is the image distance.

a Make v the subject of the formula.

b Given a focal length of 8 cm, find the image distance for the following object distances: **i** 50 cm **ii** 30 cm.

10 According to the theory of relativity by Einstein, the mass of a particle is given by the formula $m = \dfrac{m_0}{\sqrt{1 - (\frac{v}{c})^2}}$, where m_0 is the mass of the particle at rest, v is the velocity of the particle and c is the velocity of light.

a Make v the subject of the formula, (for $v > 0$).

b Find the velocity necessary to increase the mass of a particle to three times its rest mass, i.e., $m = 3m_0$. Give the value for v as a fraction of c.

c A cyclotron increased the mass of an electron to $30m_0$. With what velocity must the electron have been travelling? [**Note:** $c = 3 \times 10^8$ m/s]

J ADDING AND SUBTRACTING ALGEBRAIC FRACTIONS

Two or more algebraic fractions which are added (or subtracted) are combined into a single fraction by first obtaining the **least common denominator** (LCD).

For example, $\frac{x-1}{3} - \frac{x+3}{2}$ has LCD of 6, so we write each fraction with denominator 6.

Example 21

Write as a single fraction: **a** $2 + \frac{3}{x}$ **b** $\frac{x-1}{3} - \frac{x+3}{2}$

a
$$2 + \frac{3}{x}$$
$$= 2\left(\frac{x}{x}\right) + \frac{3}{x}$$
$$= \frac{2x+3}{x}$$

b
$$\frac{x-1}{3} - \frac{x+3}{2}$$
$$= \frac{2}{2}\left(\frac{x-1}{3}\right) - \frac{3}{3}\left(\frac{x+3}{2}\right)$$
$$= \frac{2(x-1)-3(x+3)}{6}$$
$$= \frac{2x-2-3x-9}{6}$$
$$= \frac{-x-11}{6}$$

EXERCISE J

1 Write as a single fraction:

a $3 + \frac{x}{5}$ **b** $1 + \frac{3}{x}$ **c** $3 + \frac{x-2}{2}$

d $3 - \frac{x-2}{4}$ **e** $\frac{2+x}{3} + \frac{x-4}{5}$ **f** $\frac{2x+5}{4} - \frac{x-1}{6}$

Example 22

Write $\frac{3x+1}{x-2} - 2$ as a single fraction.

$$\frac{3x+1}{x-2} - 2$$
$$= \left(\frac{3x+1}{x-2}\right) - 2\left(\frac{x-2}{x-2}\right) \quad \{\text{as } (x-2) \text{ is the LCD}\}$$
$$= \frac{(3x+1)-2(x-2)}{x-2}$$
$$= \frac{3x+1-2x+4}{x-2}$$
$$= \frac{x+5}{x-2}$$

2 Write as a single fraction:

a $1+\dfrac{3}{x+2}$ **b** $-2+\dfrac{3}{x-4}$ **c** $-3-\dfrac{2}{x-1}$

d $\dfrac{2x-1}{x+1}+3$ **e** $3-\dfrac{x}{x+1}$ **f** $-1+\dfrac{4}{1-x}$

3 Write as a single fraction:

a $\dfrac{3x}{2x-5}+\dfrac{2x+5}{x-2}$ **b** $\dfrac{1}{x-2}-\dfrac{1}{x-3}$

c $\dfrac{5x}{x-4}+\dfrac{3x-2}{x+4}$ **d** $\dfrac{2x+1}{x-3}-\dfrac{x+4}{2x+1}$

K CONGRUENCE AND SIMILARITY

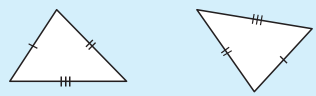
CONGRUENCE

Two triangles are **congruent** if they are identical in every respect apart from position, i.e., they have the same shape and size.

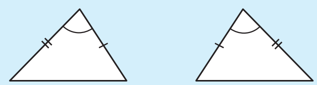
There are four acceptable tests for **congruence of two triangles**.

Two triangles are congruent if one of the following is true:

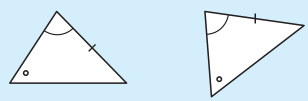
- corresponding sides are equal (**SSS**)



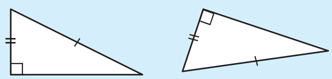
- two sides and the included angle are equal (**SAS**)



- two angles and a pair of corresponding sides are equal (**AAcorS**)

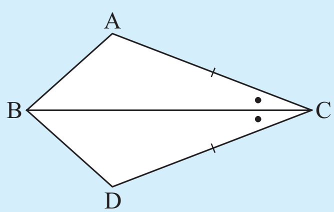


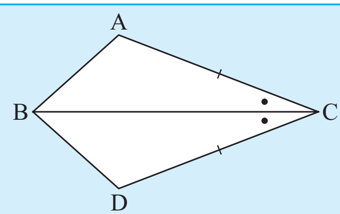
- for right angled triangles, the hypotenuses and one pair of sides are equal (**RHS**).



Example 23

Explain why ΔABC and ΔDBC are congruent:





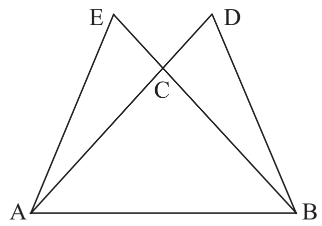
Δ's ABC and DBC are congruent (SAS) as:

- AC = DC
- $\measuredangle$ACB = $\measuredangle$DCB, and
- BC is common to both.

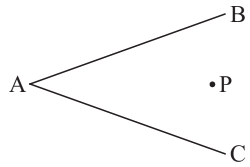
EXERCISE K.1

1 Triangle ABC is isosceles with AC = BC. BC and AC are produced to E and D respectively so that CE = CD.

Prove that AE = BD.

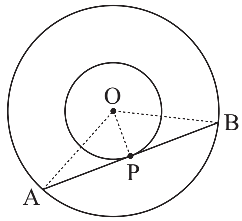


2 Point P is equidistant from both AB and AC. Use congruence to show that P lies on the bisector of ∡BAC.



3 Two concentric circles are drawn. At P on the inner circle a tangent is drawn and it meets the other circle at A and B.

Use triangle congruence to prove that P is the midpoint of AB.

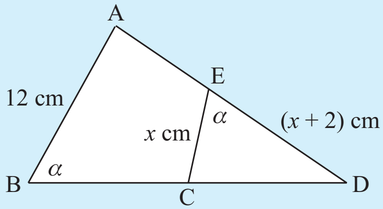


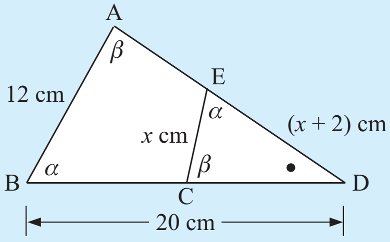
SIMILARITY

Two triangles are **similar** if one is an enlargement of the other.
Consequently, similar triangles are **equiangular**.
Similar triangles have corresponding sides in the same **ratio**.

Example 24

Establish that a pair of triangles is similar and find x if BD = 20 cm:





α	β	•	
-	$x+2$	x	small Δ
-	20	12	large Δ

The triangles are equiangular and hence similar.

$$\therefore \quad \frac{x+2}{20} = \frac{x}{12} \quad \{\text{same ratio}\}$$

$$\therefore \quad 12(x+2) = 20x$$

$$\therefore \quad 12x + 24 = 20x$$

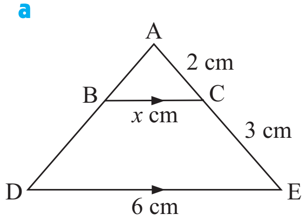
$$\therefore \quad 24 = 8x$$

$$\therefore \quad x = 3$$

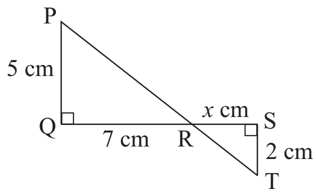
EXERCISE K.2

1 In the following, establish that a pair of triangles is similar, and find x:

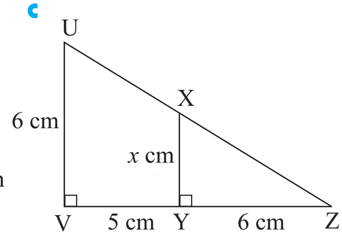
a



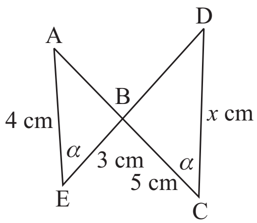
b



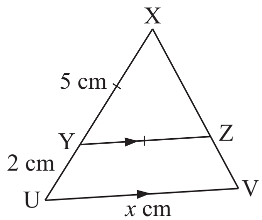
c



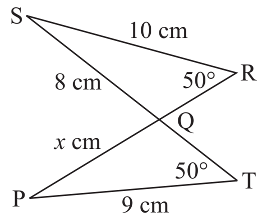
d



e



f



2 A father and son are standing side-by-side. How tall is the son if the father is 1.8 m tall and casts a shadow 3.2 m long, while his son's shadow is 2.4 m long?

EXERCISE A

1 **a** $\sqrt{15}$ **b** 3 **c** 4 **d** 12 **e** 42 **f** $\sqrt{6}$ **g** $\sqrt{2}$ **h** $\sqrt{6}$

2 **a** $5\sqrt{2}$ **b** $-\sqrt{2}$ **c** $2\sqrt{5}$ **d** $8\sqrt{5}$ **e** $-2\sqrt{5}$
f $9\sqrt{3}$ **g** $-3\sqrt{6}$ **h** $3\sqrt{2}$

3 **a** $2\sqrt{2}$ **b** $2\sqrt{3}$ **c** $2\sqrt{5}$ **d** $4\sqrt{2}$ **e** $3\sqrt{3}$ **f** $3\sqrt{5}$
g $4\sqrt{3}$ **h** $3\sqrt{6}$ **i** $5\sqrt{2}$ **j** $4\sqrt{5}$ **k** $4\sqrt{6}$ **l** $6\sqrt{3}$

4 **a** $2\sqrt{3}$ **b** $8\sqrt{2}$ **c** $5\sqrt{6}$ **d** $10\sqrt{3}$ **e** $3\sqrt{3}$ **f** $-\sqrt{2}$

5 **a** $\frac{\sqrt{2}}{2}$ **b** $2\sqrt{3}$ **c** $\frac{7\sqrt{2}}{2}$ **d** $2\sqrt{5}$ **e** $5\sqrt{2}$ **f** $3\sqrt{6}$
g $4\sqrt{3}$ **h** $\frac{5\sqrt{7}}{7}$ **i** $2\sqrt{7}$ **j** $\sqrt{6}$

EXERCISE B

1 **a** 2.59×10^2 **b** 2.59×10^5 **c** 2.59×10^0
d 2.59×10^{-1} **e** 2.59×10^{-4} **f** 4.07×10^1
g 4.07×10^3 **h** 4.07×10^{-2} **i** 4.07×10^5
j 4.07×10^8 **k** 4.07×10^{-5}

2 **a** 1.495×10^{11} m **b** 3×10^{-4} mm **c** 1×10^{-3} mm
d 1.5×10^7 °C **e** 3×10^5

3 **a** 4000 **b** 500 **c** 2100 **d** 78 000 **e** 380 000
f 86 **g** 43 300 000 **h** 60 000 000

4 **a** 0.004 **b** 0.05 **c** 0.0021 **d** 0.000 78
e 0.000 038 **f** 0.86 **g** 0.000 000 433 **h** 0.000 000 6

5 **a** 0.000 000 9 m **b** 6 130 000 000 **c** 100 000 light years
d 0.000 01 mm

6 **a** 1.64×10^{10} **b** 4.12×10^{-3} **c** 5.27×10^{-18}
d 1.36×10^2 **e** 2.63×10^{-6} **f** 1.73×10^9

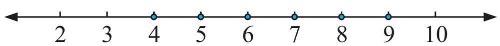
7 **a** 1.30×10^5 km **b** 9.07×10^5 km **c** 9.47×10^7 km

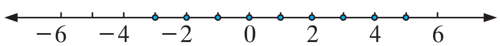
8 **a** 1.8×10^{10} m **b** 2.59×10^{13} m **c** 9.47×10^{15} m

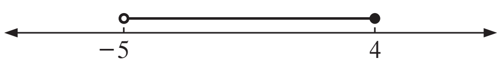
EXERCISE C

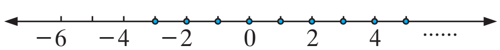
1 **a** The set of all x such that x is greater than 5.
b The set of all x such that x is less than or equal to 3.
c The set of all y such that y lies between 0 and 6.
d The set of all x such that x is greater than or equal to 2, but less than or equal to 4.
e The set of all t such that t lies between 1 and 5.
f The set of all n such that n is less than 2 or greater than or equal to 6.

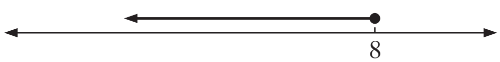
2 **a** $\{x: x > 2\}$ **b** $\{x: 1 < x \leqslant 5\}$
c $\{x: x \leqslant -2 \text{ or } x \geqslant 3\}$ **d** $\{x: x \in Z, -1 \leqslant x \leqslant 3\}$
e $\{x: x \in Z, 0 \leqslant x \leqslant 5\}$ **f** $\{x: x < 0\}$

3 **a**


b


c


d


e


EXERCISE D

1 **a** $10x - 10$ **b** $9x$ **c** $5x + 5y$ **d** $8 - 8x$ **e** $12ab$
f cannot be simplified

2 **a** $22x + 35$ **b** $16 - 6x$ **c** $4a - 3b$
d $3x^3 - 16x^2 + 11x - 1$

3 **a** $18x^3$ **b** $\frac{a}{3b}$ **c** $4x^2$ **d** $24a^{10}$

EXERCISE E

1 **a** $x = 10$ **b** $x > 6$ **c** $x = \frac{4}{5}$ **d** $x = 51$ **e** $x < -10$
f $x = 14$ **g** $x \leqslant -9$ **h** $x = 18$ **i** $x = \frac{2}{3}$

2 **a** $x = 5, y = 2$ **b** $x = \frac{22}{3}, y = \frac{8}{3}$ **c** $x = -2, y = 5$
d $x = \frac{45}{11}, y = -\frac{18}{11}$ **e** no solution **f** $x = 66, y = -84$

EXERCISE F

1 **a** 16 **b** −6 **c** 16 **d** 18 **e** −2 **f** 2

2 **a** 2 **b** 3 **c** 6 **d** 6 **e** 5 **f** −1 **g** 1 **h** 5
i 4 **j** 4 **k** 2 **l** 2

3 **a** $x = \pm 3$ **b** no solution **c** $x = 0$ **d** $x = 4$ or -2
e $x = -1$ or 7 **f** no solution **g** $x = 1$ or $\frac{1}{3}$
h $x = 0$ or 3 **i** $x = -2$ or $\frac{14}{5}$

EXERCISE G

1 **a** $2x^2 + 5x + 3$ **b** $3x^2 + 10x + 8$ **c** $10x^2 + x - 2$
d $3x^2 + x - 10$ **e** $-6x^2 + 17x + 14$ **f** $-6x^2 - 13x + 5$
g $15x^2 + 11x - 12$ **h** $15x^2 - 11x + 2$ **i** $2x^2 - 17x + 21$
j $4x^2 - 16x + 15$ **k** $-x^2 - 3x - 2$ **l** $-4x^2 - 2x + 6$

2 **a** $x^2 - 36$ **b** $x^2 - 64$ **c** $4x^2 - 1$ **d** $9x^2 - 4$
e $16x^2 - 25$ **f** $25x^2 - 9$ **g** $9 - x^2$ **h** $49 - x^2$
i $49 - 4x^2$ **j** $x^2 - 2$ **k** $x^2 - 5$ **l** $4x^2 - 3$

3 **a** $x^2 + 10x + 25$ **b** $x^2 + 14x + 49$ **c** $x^2 - 4x + 4$
d $x^2 - 12x + 36$ **e** $x^2 + 6x + 9$ **f** $x^2 + 10x + 25$
g $x^2 - 22x + 121$ **h** $x^2 - 20x + 100$ **i** $4x^2 + 28x + 49$
j $9x^2 + 12x + 4$ **k** $4x^2 - 20x + 25$ **l** $9x^2 - 42x + 49$

4 **a** $y = 2x^2 + 10x + 12$ **b** $y = 3x^2 - 6x + 7$
c $y = -x^2 + 6x + 7$ **d** $y = -x^2 - 4x - 15$
e $y = 4x^2 - 24x + 20$ **f** $y = -\frac{1}{2}x^2 - 4x - 14$
g $y = -5x^2 + 35x - 30$ **h** $y = \frac{1}{2}x^2 + 2x - 4$
i $y = -\frac{5}{2}x^2 + 20x - 40$

5 **a** $2x^2 + 12x + 19$ **b** $3x^2 + 3x - 16$ **c** $-x^2 + 6x - 6$
d $-x^2 - x + 25$ **e** $2x^2 - 16x + 33$ **f** $-3x + 4$
g $7x + 8$ **h** $7x^2 + 18x + 12$ **i** $-x^2 + 19x - 32$
j $7x^2 - 16x + 2$

EXERCISE H

1 **a** $3x(x+3)$ **b** $x(2x+7)$ **c** $2x(2x-5)$ **d** $3x(2x-5)$
e $(3x-5)(3x+5)$ **f** $(4x+1)(4x-1)$ **g** $2(x-2)(x+2)$
h $3(x+\sqrt{3})(x-\sqrt{3})$ **i** $4(x+\sqrt{5})(x-\sqrt{5})$ **j** $(x-4)^2$
k $(x-5)^2$ **l** $2(x-2)^2$ **m** $(4x+5)^2$ **n** $(3x+2)^2$
o $(x-11)^2$

2 **a** $(x+8)(x+1)$ **b** $(x+4)(x+3)$ **c** $(x-9)(x+2)$
d $(x+7)(x-3)$ **e** $(x-6)(x-3)$ **f** $(x+3)(x-2)$
g $-(x-2)(x+1)$ **h** $3(x-11)(x-3)$ **i** $-2(x+1)^2$
j $2(x+5)(x-2)$ **k** $2(x-8)(x+3)$ **l** $-2(x-6)(x-1)$
m $-3(x-1)^2$ **n** $-(x+1)^2$ **o** $-5(x-4)(x+2)$

3 **a** $(2x-3)(x+4)$ **b** $(3x+1)(x-2)$ **c** $(7x-2)(x-1)$
d $(3x-2)(2x+1)$ **e** $(2x-3)(2x+1)$ **f** $(5x-3)(2x+1)$
g $(2x+1)(x-6)$ **h** $(3x+7)(x-4)$ **i** $(4x+3)(2x-1)$
j $(5x+3)(2x-3)$ **k** $(3x-1)(x+8)$ **l** $(3x+2)(2x+1)$
m $-2(2x+3)(x-1)$ **n** $(6x+1)(2x-3)$
o $-3(2x+7)(x-2)$ **p** $-(3x-2)(3x-5)$

q $(4x-9)(2x+3)$ **r** $(4x+3)(3x+1)$
s $(6x+1)(2x+3)$ **t** $(5x-4)(3x-2)$
u $(7x+5)(2x-3)$

4 **a** $(x+4)(2x+1)$ **b** $(2-x)(5-3x)$ **c** $3(x+2)(2x+7)$
d $4(x+5)(2x+11)$ **e** $2x(x+3)$ **f** $5(x+3)$
g $(x-2)(3x+26)$ **h** $(x-1)(2x-1)$

5 **a** $(x+7)(x-1)$ **b** $(x+1)(3-x)$ **c** $12(x+1)$
d $-4x(x+4)$ **e** $(3x+2)(x+4)$ **f** $h(2x+h)$
g $-12(x+1)$ **h** $-5(3x-4)(x-4)$ **i** $-3(x+9)(5x+9)$

EXERCISE I

1 **a** $x=b-a$ **b** $x=\frac{b}{a}$ **c** $x=\frac{d-a}{2}$ **d** $x=t-c$
e $x=\frac{20-2y}{5}$ **f** $x=\frac{12-3y}{2}$ **g** $x=\frac{d-3y}{7}$
h $x=\frac{c-by}{a}$ **i** $x=\frac{y-c}{m}$

2 **a** $z=\frac{b}{ac}$ **b** $z=\frac{a}{d}$ **c** $z=\frac{2d}{3}$

3 **a** $a=\frac{F}{m}$ **b** $r=\frac{C}{2\pi}$ **c** $d=\frac{V}{lh}$ **d** $K=\frac{b}{A}$

4 **a** $r=\sqrt{\frac{A}{\pi}}$ **b** $x=\sqrt[5]{aN}$ **c** $r=\sqrt[3]{\frac{3V}{4\pi}}$ **d** $x=\sqrt[3]{\frac{n}{D}}$

5 **a** $a=d^2n^2$ **b** $l=25T^2$ **c** $a=\pm\sqrt{b^2+c^2}$ **d** $l=\frac{gT^2}{4\pi^2}$
e $a=\frac{P}{2}-b$ **f** $h=\frac{A-\pi r^2}{2\pi r}$ **g** $r=\frac{E}{I}-R$ **h** $q=p-\frac{B}{A}$

6 **a** $a=\frac{d^2}{2kb}$ **b** 1.29 **7** **a** $r=\sqrt[3]{\frac{3V}{4\pi}}$ **b** 2.122 cm

8 **a** $t=\sqrt{\frac{2S}{a}}$ **b** 15.81 sec

9 **a** $v=\frac{uf}{u-f}$ **b** **i** 9.52 cm **ii** 10.9 cm

10 **a** $v=\sqrt{c^2\left(1-\frac{{m_0}^2}{m^2}\right)}=\frac{c}{m}\sqrt{m^2-{m_0}^2}$
b $v=\frac{\sqrt{8}}{3}c$ **c** 2.998×10^8 m/s

EXERCISE J

1 **a** $\frac{15+x}{5}$ **b** $\frac{x+3}{x}$ **c** $\frac{x+4}{2}$ **d** $\frac{14-x}{4}$
e $\frac{8x-2}{15}$ **f** $\frac{4x+17}{12}$

2 **a** $\frac{x+5}{x+2}$ **b** $\frac{11-2x}{x-4}$ **c** $\frac{1-3x}{x-1}$ **d** $\frac{5x+2}{x+1}$
e $\frac{2x+3}{x+1}$ **f** $\frac{x+3}{1-x}$

3 **a** $\frac{7x^2-6x-25}{(2x-5)(x-2)}$ **b** $\frac{-1}{(x-2)(x-3)}$
c $\frac{8x^2+6x+8}{x^2-16}$ **d** $\frac{3x^2+3x+13}{(x-3)(2x+1)}$

EXERCISE K.1

1 **Hint:** Consider Δs AEC, BDC
2 **Hint:** Let M be on AB so that PM $\perp$ AB
Let N be on AC so that PN $\perp$ AC
Join PM, PN and consider the two triangles formed.
3 No hint needed.

EXERCISE K.2

1 **a** $x=2.4$ **b** $x=2.8$ **c** $x=3\frac{3}{11}$ **d** $x=6\frac{2}{3}$
e $x=7$ **f** $x=7.2$

2 1.35 m tall